

Principles of Math 12

Name: _____

Trigonometry Review Assignment:

1. Convert 100° to radians.

- A. 0.18 B. 0.57 C. 1.75 D. 5.66

2. Give the period of $y=2 \sec(x)$

- A. $\frac{1}{2\pi}$ B. $\frac{\pi}{2}$ C. π D. 2π

3. Determine the exact value of $\tan \frac{2\pi}{3}$

- A. $-\sqrt{3}$ B. $-\frac{1}{\sqrt{3}}$ C. $\frac{1}{\sqrt{3}}$ D. $\sqrt{3}$

4. Give the range of $y = -5\sin \frac{1}{2}x + 8$

- A. $3 \leq y \leq 8$ B. $3 \leq y \leq 13$ C. $-13 \leq y \leq -3$ D. $-13 \leq y \leq 13$

5. Simplify $4-8\sin^2(6x)$

- A. $\cos 12x$ B. $2\cos 6x$ C. $4\cos 6x$ D. $4\cos 12x$

6. Determine an equation of an asymptote of $y = -2 \tan x$

- A. $x=\pi/4$ B. $x=\pi/2$ C. $x=\pi$ D. $x=2\pi$

7. Solve: $\sin x = 1/x$, $0 \leq x \leq 2\pi$

- A. 0, 1.56 B. 1.11, 2.77 C. 3.44, 6.11 D. 0, 3.14, 6.28

8.

At a seaport, the water has a maximum depth of 18 m at 3:00 am. After this maximum depth, the first minimum depth of 4 m occurs at 9:30 am. Assume that the relation between the depth, h metres, and the time, t hours, is a sinusoidal function. Determine an equation for h at any time t .

A. $h = 7 \cos 2\pi \frac{(t-3)}{6.5} + 11$

B. $h = 7 \cos 2\pi \frac{(t-3)}{13} + 11$

C. $h = 11 \cos 2\pi \frac{(t-3)}{6.5} + 7$

D. $h = 11 \cos 2\pi \frac{(t-3)}{13} + 7$

9.

Point P is the intersection of the terminal arm of angle θ in standard position and the unit circle with centre $(0, 0)$. If P is in quadrant 3 and $\cos \theta = m$, determine the coordinates of P in terms of m .

A. $(-m, \sqrt{1-m^2})$

B. $(-m, -\sqrt{1-m^2})$

C. $(m, \sqrt{1-m^2})$

D. $(m, -\sqrt{1-m^2})$

10. Determine the number of solutions in the interval $0 \leq x < 2\pi$ for:
 $\sin(ax) = 0.5$, where a is an integer and $a \geq 1$

A. 2 B. $a/2$ C. a D. $2a$

11. a) Solve algebraically, giving exact values for x , where $0 \leq x < 2\pi$.
 $2\cos^2 x - \cos x = 1$

b) Give the general solution for this equation. (Solve over the set of real numbers, giving exact value solutions.)

12. Prove the identity: $\frac{\cos \theta + \cot \theta}{1 + \sin \theta} = \cot \theta$

13. Convert 120° to radians.

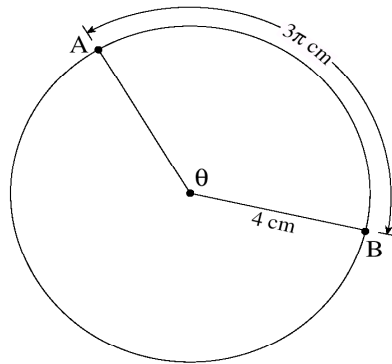
A. $2\pi/3$ B. $5\pi/6$ C. $3\pi/2$ D. $6\pi/5$

14. Determine the amplitude of $y = -2\sin 3(x - \frac{\pi}{4}) + 4$

A. -2 B. 2 C. 3 D. 4

15.

A circle has a radius of 4 cm. If the length of arc AB shown on the diagram is 3π cm, determine the measure of the central angle θ in radians.



- A. $\frac{3\pi}{4}$
- B. $\frac{4}{3\pi}$
- C. $\frac{3\pi}{2}$
- D. 3π

16. Solve: $\tan x - \cos x = -2$, $0 \leq x \leq 2\pi$

- A. 1.17, 4.10
- B. 1.97, 5.32
- C. 1.17, 1.57, 4.10, 4.71
- D. 1.57, 1.97, 4.71, 5.32

17.

Solve: $4 \cos^2 x = 3$, $0 \leq x < 2\pi$

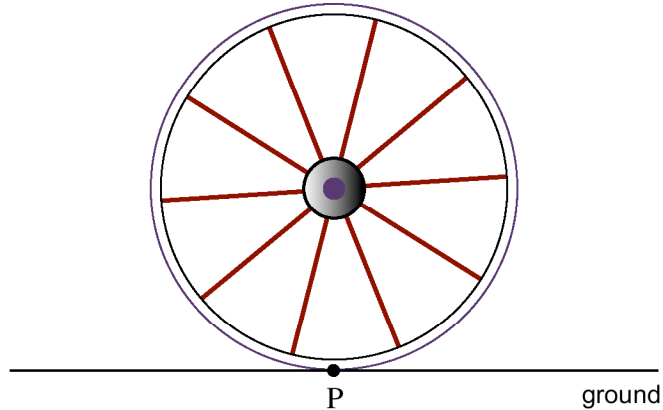
- A. $\frac{\pi}{6}, \frac{11\pi}{6}$
- B. $\frac{\pi}{3}, \frac{5\pi}{3}$
- C. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
- D. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

18. Determine an expression equivalent to $\tan \beta + \cot \beta$.

- A. 1
- B. $\sin \beta \cos \beta$
- C. $\sec \beta \csc \beta$
- D. $\sin \beta + \cos \beta$

19.

A wheel with diameter 10 cm is rolling along the ground. Point P on the edge of the wheel is on the ground as shown in the diagram at time $t = 0$ seconds. Which equation gives the height, h , of point P above the ground at time t seconds, if the wheel rotates once every 12 seconds?



- A. $h = -5 \cos \frac{\pi}{12} t$
- B. $h = -5 \cos \frac{\pi}{6} t$
- C. $h = -5 \cos \frac{\pi}{12} t + 5$
- D. $h = -5 \cos \frac{\pi}{6} t + 5$

20. The point (p, q) is the point of intersection of the terminal arm of angle β in standard position and the unit circle centered at $(0,0)$. Which expression represents $\csc \beta$?

- A. p B. q C. $1/p$ D. $1/q$

21. Which expression is equivalent to $6 \sin(8x)\cos(8x)$?

- A. $\sin(8x)$ B. $\sin(16x)$ C. $3\sin(4x)$ D. $3\sin(16x)$

22.

Determine the equations of the asymptotes of the function $y = \tan bx$, where $b > 0$.

- A. $x = \frac{n\pi}{b}$, n is an integer
- B. $x = \frac{n\pi}{2b}$, n is an integer
- C. $x = \frac{\pi}{b} + \frac{n\pi}{b}$, n is an integer
- D. $x = \frac{\pi}{2b} + \frac{n\pi}{b}$, n is an integer

23. a) Solve algebraically, giving **exact values** for x , where $0 \leq x \leq 2\pi$.

$$2\sin^2 x = \sin x$$

b) Give the general solution for this equation.

24. Give the exact value of $\cos \frac{11\pi}{6}$

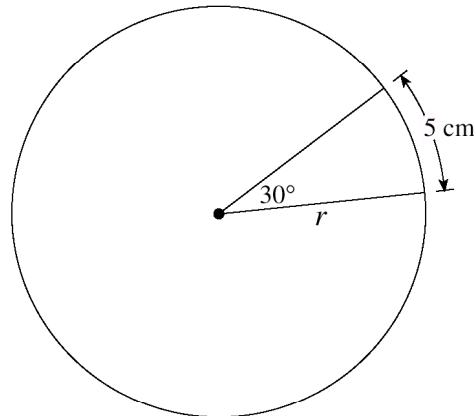
- A. $-\frac{\sqrt{3}}{2}$ B. $-\frac{\sqrt{2}}{2}$ C. $\frac{\sqrt{2}}{2}$ D. $\frac{\sqrt{3}}{2}$

25. Simplify $\frac{2\sin\theta}{\sin 2\theta}$

- A. 1 B. $\cos\theta$ C. $\csc\theta$ D. $\sec\theta$

26.

An arc of length 5 cm subtends an angle of 30° at the centre of a circle with radius r , as shown in the diagram. Determine the value of r .



- A. 4.77
B. 6.00
C. 9.55
D. 10.00

27. Determine the period of $y = \tan(\pi x)$

- A. 1 B. 2 C. $\pi/2$ D. π

28. Solve: $3\sin x = x + 1$, $0 \leq x \leq 2\pi$

- A. 0.25 B. 1.87, 2.87 C. 0.54, 1.54 D. 0.54, 1.87

29. Simplify: $\sin\left(\frac{3\pi}{2} + x\right)$

- A. $\sin x$ B. $\cos x$ C. $-\sin x$ D. $-\cos x$

30. Solve: $\sin^2 x = \sin x \cos x$, $0 \leq x < 2\pi$

- A. $x=0, \frac{\pi}{4}$ B. $x=\frac{\pi}{4}, \frac{5\pi}{4}$
C. $x=0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$ D. $x=0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

31. The terminal arm of angle ϕ in standard position passes through the point $(-2, 5)$. Determine the value of $\sec \phi$.

- A. $-\frac{\sqrt{21}}{2}$ B. $\frac{\sqrt{21}}{5}$ C. $-\frac{\sqrt{29}}{2}$ D. $\frac{\sqrt{29}}{5}$

32. Determine the range of the function: $y = b \cos(ax) - 2b$, where $a > 0, b > 0$.

- A. $b \leq y \leq 3b$ B. $-3b \leq y \leq -b$ C. $b - a \leq y \leq b + a$ D. $2b - a \leq y \leq 2b + a$

33.

Determine the general solution for: $\sin 2x = -\frac{1}{2}$

- A. $\frac{7\pi}{12} + 2n\pi, \frac{11\pi}{12} + 2n\pi$ (n is any integer)
B. $\frac{7\pi}{12} + n\pi, \frac{11\pi}{12} + n\pi$ (n is any integer)
C. $\frac{13\pi}{12} + 2n\pi, \frac{21\pi}{12} + 2n\pi$ (n is any integer)
D. $\frac{13\pi}{12} + n\pi, \frac{21\pi}{12} + n\pi$ (n is any integer)

34. A Ferris wheel has a radius of 26 m and its centre is 29 m above the ground. It rotates once every 48 seconds. Sandy gets on the Ferris wheel at its lowest point, and then the wheel starts to rotate.

a) Determine a sinusoidal equation that gives Sandy's height, h , above the ground as a function of the elapsed time, t , where h is in metres and t is in seconds.

b) Determine the first time t (in seconds), when Sandy will be 36 m above the ground.

35. Find an equivalent form for the following: $\frac{\sin x}{1 - \sin x} - \frac{\sin x}{1 + \sin x}$

A. $\tan^2 x$ B. $\cot^2 x$ C. $2\tan^2 x$ D. $2\cot^2 x$

36. Convert $\frac{5\pi}{3}$ radians to degrees. Do NOT use a calculator.

A. 60° B. 120° C. 300° D. 330°

37. Solve: $\sin 2x - \cos x = 1$, $0 \leq x \leq 2\pi$

A. 0, 5.07 B. 3.14, 4.32 C. 3.14, 4.36 D. 0.42, 1.89, 2.95, 4.21

38. Determine the exact value of $\cot \frac{-5\pi}{3}$. Do NOT use a calculator.

A. $-\frac{1}{\sqrt{3}}$ B. $\frac{1}{\sqrt{3}}$ C. $-\sqrt{3}$ D. $\sqrt{3}$

39. Determine the period of the function $f(x) = -\frac{1}{2}\sin \frac{x}{3}$.

A. $\frac{2\pi}{3}$ B. π C. 4π D. 6π

40. Solve: $2 \sin x + 1 = 0$, $0 \leq x \leq 2\pi$

A. $-\frac{\pi}{6}, -\frac{5\pi}{6}$ B. $\frac{\pi}{6}, \frac{5\pi}{6}$ C. $\frac{7\pi}{6}, \frac{11\pi}{6}$ D. $\frac{4\pi}{3}, \frac{5\pi}{3}$

41. Determine an equivalent form for the following: $\frac{\tan \theta \csc^2 \theta}{\sec^2 \theta}$

A. $\tan \theta$ B. $\cot \theta$ C. $\tan^2 \theta$ D. $\tan^3 \theta$

42. Simplify: $\cos(\pi - 2x)$

- A. $-\cos 2x$ B. $-\sin 2x$ C. $\cos 2x$ D. $\sin 2x$

43.

A wheel with radius 20 cm has its centre 30 cm above the ground. It rotates once every 15 seconds. Determine an equation for the height, h , above the ground of a point on the wheel at time t seconds if this point has a maximum height at $t = 2$ seconds.

A. $h = 20 \cos \frac{2\pi}{15}(t + 2) + 30$

B. $h = 20 \cos \frac{2\pi}{15}(t - 2) + 30$

C. $h = 30 \cos \frac{2\pi}{15}(t + 2) + 20$

D. $h = 30 \cos \frac{2\pi}{15}(t - 2) + 20$

44.

Determine a cosine equation that has the following general solution: $\frac{\pi}{2} + n\pi, \frac{\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$, where n is an integer.

A. $\cos x(2 \cos x + \sqrt{2}) = 0$

B. $\cos x(2 \cos x + \sqrt{3}) = 0$

C. $\cos x(2 \cos x - \sqrt{2}) = 0$

D. $\cos x(2 \cos x - \sqrt{3}) = 0$

45. Solve the following equation algebraically.

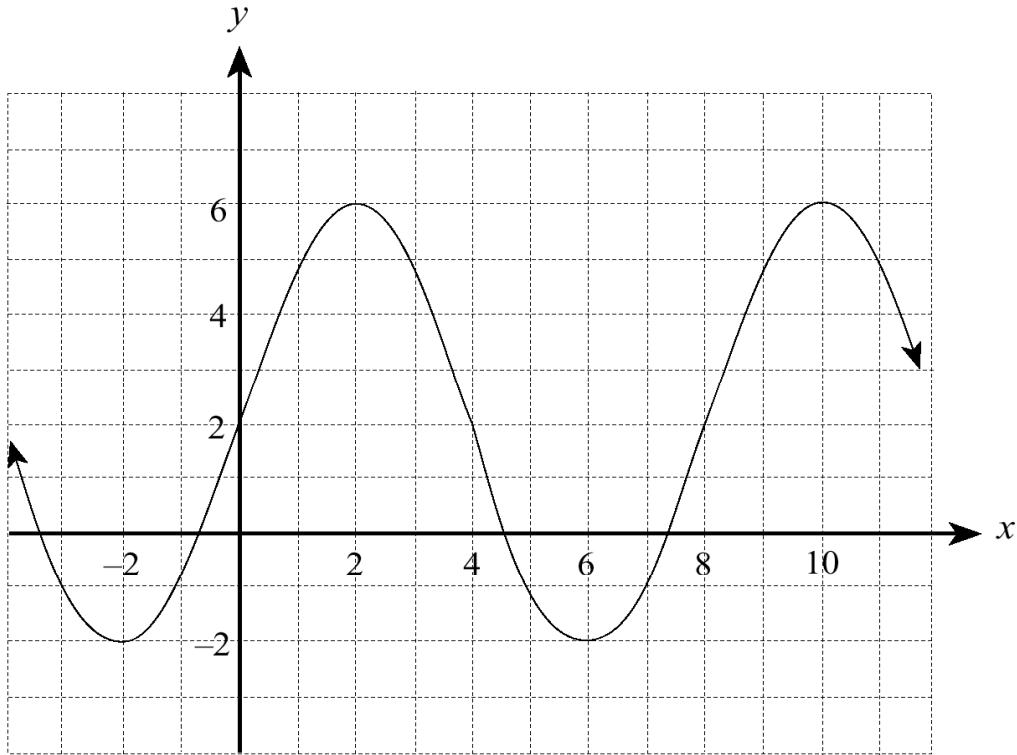
$$3\cos^2 x + \cos x - 2 = 0, \quad 0 \leq x \leq 2\pi$$

46. Prove the identity: $(\csc \theta - \sin \theta) \tan \theta = \frac{\sin 2\theta}{2 \sin \theta}$

47. Evaluate to 2 decimal places: $\sec 0.89$

Ans: _____

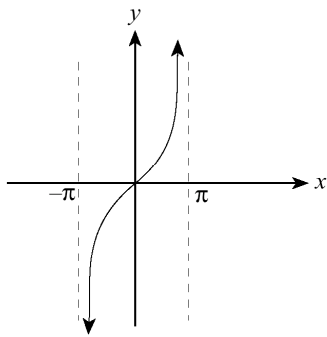
48. Determine the amplitude of the following graph:



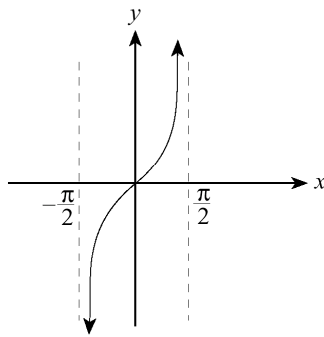
- A. 2 B. 4 C. 6 D. 8

49. Which graph shows one period of $y = \tan x$?

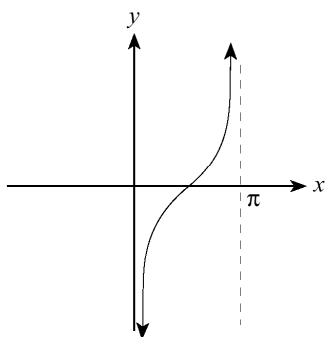
A.



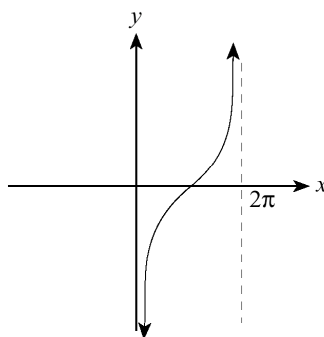
B.



C.



D.



50. Convert m radians to degrees.

- A. $\frac{\pi}{180m}^\circ$ B. $\frac{\pi m}{180}^\circ$ C. $\frac{180}{\pi m}^\circ$ D. $\frac{180m}{\pi}^\circ$

51. Simplify $\cos(\frac{3\pi}{2} + \beta)$

- A. $\sin \beta$ B. $-\sin \beta$ C. $\cos \beta$ D. $-\cos \beta$

52. Determine the phase shift of the function $f(x) = -3\sin(2x - \frac{\pi}{6}) + \frac{\pi}{2}$

- A. $\pi/12$ B. $\pi/6$ C. $\pi/3$ D. $\pi/2$

53. Which expression is equivalent to $(\sin^2\beta - \cos^2\beta)^2 - \sin^2 2\beta$?

- A. $-2\sin^2 2\beta$ B. $2\sin^2 2\beta$ C. $-\cos 4\beta$ D. $\cos 4\beta$

54. Solve for x : $2\tan^2 x - 5\tan x = 3$, where $0 \leq x \leq 2\pi$ (accurate to 2 decimal places)

55.

For what value of x is the following expression undefined?

$$\frac{\sin x}{1 + \cos x}, \text{ where } 0 \leq x < 2\pi$$

- A. 0
B. $\frac{\pi}{2}$
C. π
D. $\frac{3\pi}{2}$

56. Simplify: $\frac{\sqrt{\sec^2 - 1}}{\sqrt{\csc^2 - 1}}$

- A. $\tan^2 x$ B. $\cot^2 x$ C. $\tan^4 x$ D. $\cot^4 x$

57.

Determine the value of $\sec \theta$ if $\cot \theta = -a$, where $a > 0$ and $\sin \theta < 0$.

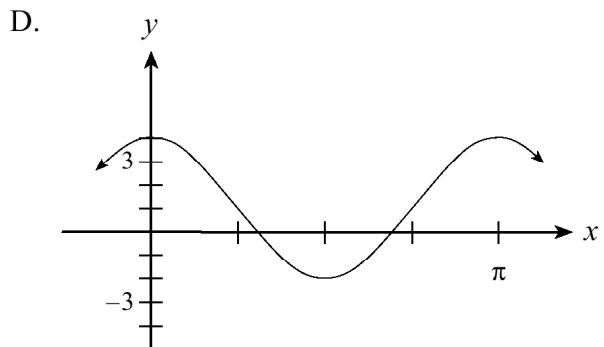
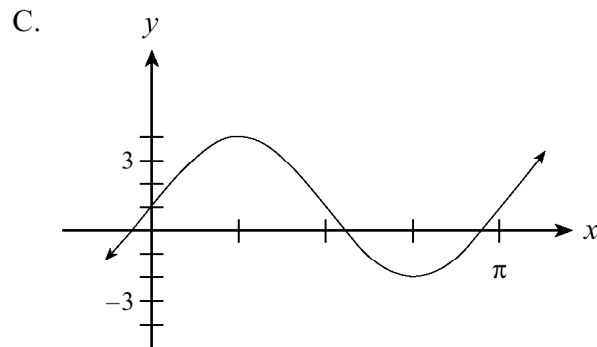
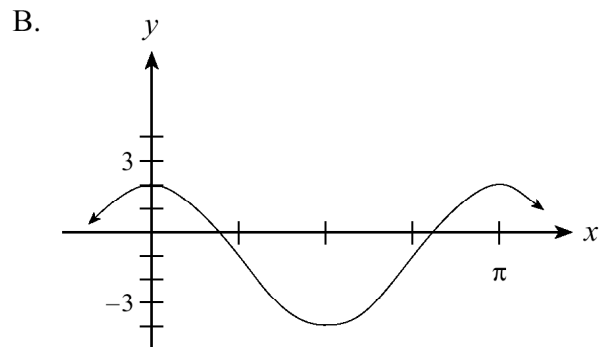
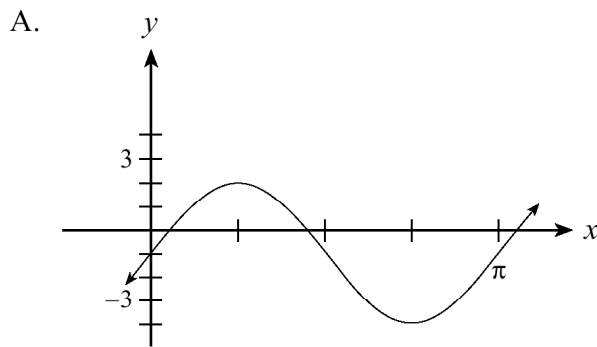
A. $\frac{\sqrt{a^2 + 1}}{a}$

B. $-\frac{\sqrt{a^2 + 1}}{a}$

C. $\frac{a+1}{a}$

D. $-\frac{a+1}{a}$

58. Which of the following represents the graph of $y = 3\sin 2x - 1$?



59. How many solutions does $\sin^2 x = 1/3$ have over the interval $0 \leq x \leq 2\pi$?

A. 1

B. 2

C. 3

D. 4

60.

Which expression is equivalent to $\frac{2 \tan \theta \cot 2\theta}{1 + \tan \theta}$?

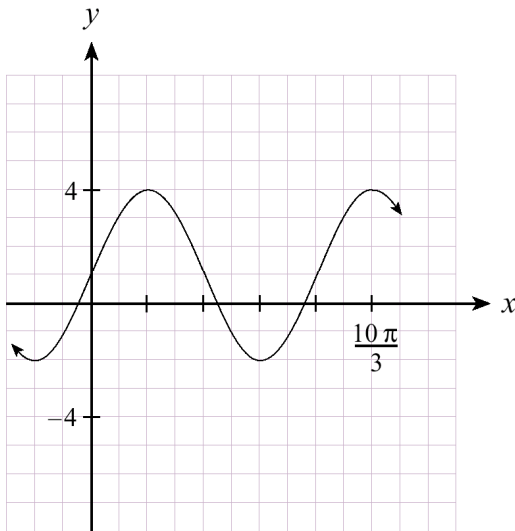
A. $1 - \tan \theta$

B. $\frac{1}{1 - \tan \theta}$

C. $\frac{1}{1 + \tan \theta}$

D. $\frac{1}{\tan \theta}$

61. The function $y = a \cos b(x - c) + d$ is graphed below. Determine b .



A. $3/5$

B. $3/4$

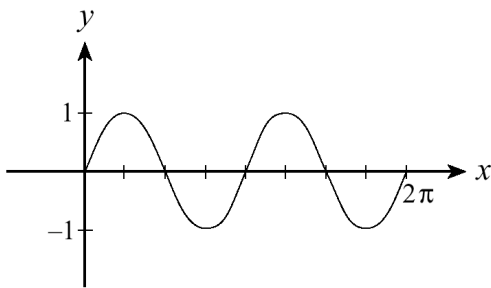
C. $\frac{8\pi}{3}$

D. $\frac{10\pi}{3}$

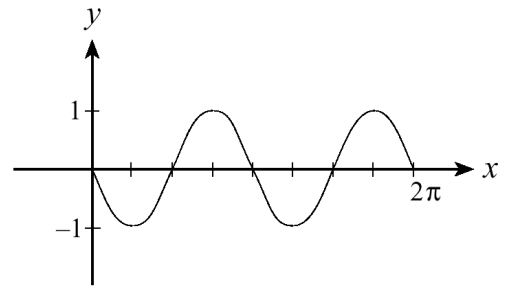
62.

Which of the following is the graph of $y = -\sin 2x$, for $0 \leq x \leq 2\pi$?

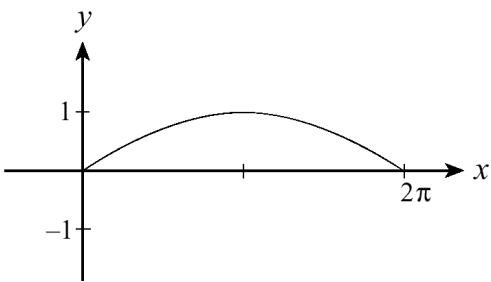
A.



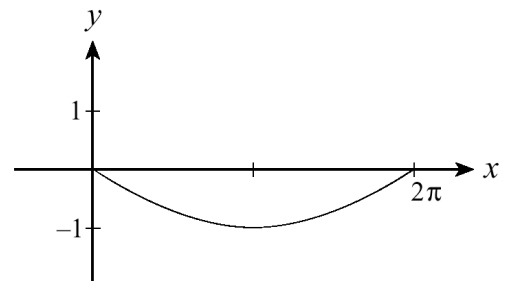
B.



C.

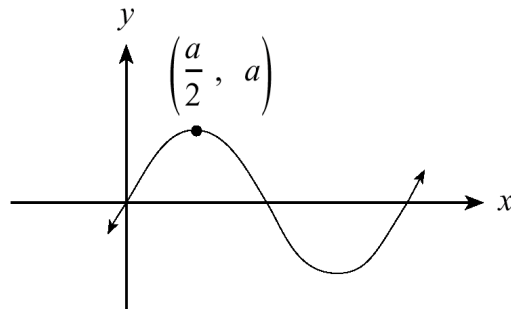


D.



63.

Determine the equation of the following sine curve.



A. $y = a \sin \frac{\pi}{a} x$

B. $y = a \sin \frac{a}{\pi} x$

C. $y = \frac{a}{2} \sin \frac{\pi}{a} x$

D. $y = \frac{a}{2} \sin \frac{2a}{\pi} x$

64.

A cosine curve has a maximum point at (3, 20) and the nearest minimum point to the right of this point is (8, 4). Which of the following is an equation for this curve?

A. $y = 8 \cos \frac{2\pi}{5}(x + 3) + 12$

B. $y = 8 \cos \frac{2\pi}{5}(x - 3) + 12$

C. $y = 8 \cos \frac{\pi}{5}(x + 3) + 12$

D. $y = 8 \cos \frac{\pi}{5}(x - 3) + 12$

65.

If the point (1, 2) lies on the terminal arm of angle θ in standard position, determine the value of $\cos(\pi + \theta)$.

A. $\frac{-2}{\sqrt{5}}$

B. $\frac{-1}{\sqrt{5}}$

C. $\frac{1}{\sqrt{5}}$

D. $\frac{2}{\sqrt{5}}$

Scholarship Questions! Nasty, with big fangs and sharp teeth! Be careful!

66. Prove the identity: $\frac{\sin 2x}{1 - \cos 2x} = 2 \csc 2x - \tan x$

67. Solve algebraically for x : $4 \sin^2 x = 3 \tan^2 x - 1$, $0 \leq x \leq 2\pi$. Give your answers as exact solutions.

68. Solve algebraically: $\sin 2x = 2 \cos x \cos 2x$, $0 \leq x \leq 2\pi$. Give your answer to two decimal places.

69. Change the equation $y = 6 \sin x \cos^3 x + 6 \sin^3 x \cos x - 3$ to the form $y = A \sin Bx + D$, where A, B, and D are constants.

70. Solve algebraically: $\sec x + \tan^2 x - 3\cos x = 2$, $0 \leq x \leq 2\pi$. (Accurate to 2 decimal places).

71. a) Prove the identity: $\frac{1 - \sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = \frac{1}{1 + \sec x}$

b) Solve the following equation for x , $0 \leq x \leq 2\pi$.

$$\frac{1 - \sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = -\frac{1}{3}, \text{ accurate to at least 2 decimal places.}$$

72. If the infinite geometric series $1 - \sin x + \sin^2 x - \sin^3 x + \dots$ has a sum of 2, what is the smallest positive value of x ? Give your answer in radians.

73. Prove the following identity: $\frac{1 + \cos 2\theta}{2} = \cos^2 \theta$

74. Prove the following identity: $\cos 4\theta = 8\cos^4 \theta - 8\cos^2 \theta + 1$

75. Solve algebraically. Give your answer to 2 decimal places.

$$3 \sin x = 4 + 4 \csc x \quad 0 \leq x \leq 2\pi$$

76. a) For what values of x in the interval $0 \leq x \leq 2\pi$ is the following equation **undefined**?

$$\log_3\left(\frac{\cos x}{2}\right) - \log_3(\cos^2 x) = 1$$

b) Solve the above equation for x ($0 \leq x \leq 2\pi$). (Accurate to 2 decimal places)

Answers:

1. c
2. d
3. a
4. b
5. d
6. b
7. b
8. b
9. d
10. d
11. a) $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ or 0

$$b) x = \frac{2\pi}{3} + 2n\pi$$

$$x = \frac{4\pi}{3} + 2n\pi$$

$$x = 0 + 2n\pi$$

12. see solution sheet
13. a
14. b
15. a
16. b
17. c
18. c
19. d
20. d
21. d
22. d
23. a) $x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

$$b) x = n\pi, n \in I$$

$$x = \frac{\pi}{6} + 2n\pi, n \in I$$

$$x = \frac{5\pi}{6} + 2n\pi, n \in I$$

24. d

25. d
26. c
27. a
28. d
29. d
30. d
31. c
32. b
33. b
34. a)
$$h(t) = -26\cos\left(\frac{\pi}{24}t\right) + 29$$

b) 14.1 s
35. c
36. c
37. c
38. b
39. d
40. c
41. b
42. a
43. b
44. d
45. $x=0.84, 3.14, 5.44$
46. see solution sheet
47. 1.59
48. b
49. b
50. d
51. a
52. a
53. d
54. $x=1.25, 2.68, 4.39, 5.82$
55. c
56. a
57. a
58. a
59. d

60. a
61. b
62. b
63. a
64. d
65. b
66. see solution sheet
67. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
68. $x=0.52, 1.57, 2.62, 4.71$
69. $y = 3\sin(2x) - 3$
70. $x=.96, 2.19, 3.14, 4.10, 5.33$
71. a) see solution sheet
b) $x=1.82, 4.46$
72. $x = \frac{7\pi}{6}$
73. see solution sheet
74. see solution sheet
75. $x=3.87, 5.55$
76. a) undefined for
 $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$ b) 1.40, 4.88

Note to teachers:

The questions here come from a variety of sources. Some come from Alberta provincial exams, or are based on questions from those documents. The scholarship questions at the end come from the BC scholarship exams from 1991 to 1996. Most of the multiple choice questions are based on provincial exams from 1994-1996, but I have tried to change the numbers where formatting was not too large an issue. I haven't put many trig identities on the written sections. If you want more trig identities, go to the BCAMT website and download Mark Garneau's outstanding list of all of the trig identities from provincial exams (it goes back a dozen years or so).

I generally hand this out at the beginning of the unit (including the answer key), and I collect it the day of the test. I flip through the booklet just to see if there is writing on each page, and I give the students a few marks. During the unit, I have a few photocopied solution manuals (showing all my steps) floating around the class as well. Students can sign them out and take them home if they wish.

If you find any errors in the answer key, or have any suggestions that I could add, feel free to email me at kdueck@sd42.ca and I'll be happy to reply.

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PS Thanks to Gretchen McConnell for helping with error checking my answer keys! It's been much appreciated!